

1-D POLLUTANT MIGRATION IN SOILS OF FINITE DEPTH

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ABSTRACT: A technique for the analysis of 1-D pollutant migration through a clay layer of finite depth is presented. This formulation includes dispersive and advective transport in the clay as well as geochemical reactions and permits consideration of the depletion of contaminant in the landfill with time as well as the effect of ground-water flow in a permeable stratum beneath the clay layer. A limited parametric study is presented to illustrate the effect of considering these factors in the analysis. It is shown that for most practical situations the concentration of contaminant within the ground water beneath the landfill will reach a peak value at a specific time and will then decrease with subsequent time. It is shown that the magnitude of this peak concentration and the time required for it to occur are highly dependent upon the mass of contaminant within the landfill and the sorption capacity of the clay. Other important factors which are examined include the thickness of the clay layer, the advection velocity (relative to the dispersivity), and the ground-water flow velocity in any permeable strata beneath the clay layer. The implications of these results for optimizing the design of clay liners is then discussed.

INTRODUCTION

Geotechnical engineers are becoming increasingly involved with the problems of pollutant migration through soil. This involvement arises from concern regarding the contamination of the ground-water system by toxic substances which have been, or are being, stored in landfills and lagoons. Often, these disposal sites are located in a clay deposit or have a clay liner. Since the movement of pollutant through relatively impermeable soils is quite slow, the time required for severe contamination of the surrounding ground water may range from several to hundreds of years. The design of these disposal sites should, and now generally does, require consideration of the likely contamination of the surrounding ground-water system in both the short and long term.

Optimal design of waste-disposal facilities requires an understanding of the fundamental mechanisms and the material properties in the appropriate chemical and hydraulic environment, as well as the availability of mathematical models which can be used to make quantitative predictions of liner performance. This paper is concerned with the latter of these requirements.

The key factors governing contaminant migration are advection, dispersion, and chemical reaction where, for many applications involving clay liners or clay deposits, the advective transport is small compared with that due to dispersion (e.g., Refs. 4, 9 and others). Several inves-

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tigators have examined this general problem using both analytical and numerical techniques (1,5). However, existing solutions do not appear to encompass a number of important practical situations.

In this paper, a technique will be developed for the analysis of one-dimensional contaminant transport of a single solute in a layer of finite thickness. While various aspects of the problem examined herein have been recognized in the literature, the present approach differs from previous work in that it considers the combination of the effects of advection, diffusion-dispersion, and chemical retardation in one model with a finite quantity of pollutant in the landfill and moving ground-water beneath the clay deposit-liner. The effect of considering these different factors is demonstrated by means of a parametric study and the implications for design are discussed. Problems involving two-dimensional flow, layering, and multiple interacting solute transport will be examined in subsequent publications.

THEORY

Governing Equations.—Assuming that the physical processes of molecular diffusion and mechanical dispersion can both be treated as Fickian type spreading mechanisms, the two effects can be lumped together in terms of a single "coefficient of hydrodynamic dispersion" D . It is found that for one-dimensional transport in the z direction, the mass flux f is given by

$$f = nvc - nD \frac{\partial c}{\partial z} \dots \dots \dots (1)$$

in which the first and second terms on the right-hand side represent the advective and diffusive-dispersive transport, respectively; n = the effective porosity; c = the concentration (mass per unit volume of fluid); and v = the average linearized seepage velocity. The velocity $v_a = nv$ will be used to describe advection when analyzing the results. The quantity v_a is variously called the advective velocity, the superficial velocity, the apparent velocity, the Darcy velocity, or the discharge velocity. For the sake of uniformity and simplicity of presentation, the term advective velocity will be used throughout this paper.

Consideration of mass balance gives,

$$-\frac{\partial f}{\partial z} = n \frac{\partial c}{\partial t} + g(c, t) \dots \dots \dots (2)$$

in which the function $g(c, t)$ describes the geochemical reaction. For equilibrium controlled ion exchange, where the concentration of one exchange ion is relatively low, the absorption of this species can often be approximated by a linear relationship between the contaminant absorbed and the concentration in the pore fluid, and therefore

$$g(c, t) = \rho K \frac{\partial c}{\partial t} \dots \dots \dots (3)$$

in which ρ = the bulk density of solid; and K = the distribution coefficient which may be determined experimentally for a given soil and solute species.

The linear relationship implied by Eq. 3 is a simplification of the real situation; however, provided that the parameter K is determined over a representative range of concentrations, this approach can be shown to be adequate for many practical applications (the use of this linear approximation is analogous to the use of secant elastic moduli in settlement predictions). The distribution coefficient K may be estimated from backanalysis of the results of laboratory column tests (21) or using batch techniques (e.g., 11, 12, 14).

Combining Eqs. 2 and 3 gives the equation

$$(n + \rho K) \frac{\partial c}{\partial t} = - \frac{\partial f}{\partial z} \dots \dots \dots (4)$$

The equations developed in this paper are for saturated flow. However, it is likely that the approach could also be approximately used for situations where there is advective-dispersive transport through partially saturated soil provided that the relevant parameters were determined under conditions similar to that anticipated in the field.

Boundary Conditions.—Landfills are of finite extent and have a limited active life. Typically, landfills are constructed in cells. Decomposition will commence immediately after construction of a particular cell; for a period of time (which will depend upon the specific conditions) the concentration of a particular contaminant in the leachate will increase until a maximum is reached. This process may take several years and can be modeled, using superposition, if the details are known. However, if the landfill is constructed on a clay deposit or clay liner, the time to reach peak concentrations is often small compared with the time scale imposed by the slow pollutant migration through the clay. Thus, in many practical situations, it can be assumed that the most hazardous pollutant has a maximum concentration, c_o , shortly after construction (time zero) and that this concentration will then decrease with time as leachate is transmitted through the soil. If the equivalent height of leachate in the landfill is H_f (the equivalent height of leachate equals the volume of leachate divided by the plan area of the landfill), then the surface concentration of any time t (after the peak concentration is reached) is given by

$$c(t) = c_o - \frac{1}{H_f} \int_0^t f_o(c, \tau) d\tau \dots \dots \dots (5)$$

in which $f_o(c, \tau)$ is the surface flux at $z = 0$ and is defined by Eq. 1.

The height of leachate H_f will probably vary somewhat, due to seasonal fluctuations. However, since these changes are rapid compared with the time scale of the problem, it is reasonable to use an average value of H_f in Eq. 5. The case most frequently investigated by previous workers (e.g., Refs. 3, 13, 16, 17, 19) is that where the surface concentration remains constant (i.e., is independent of time). It follows from Eq. 5 that this can only occur if H_f is very large (denoted in this paper by $H_f = \infty$). This assumption of a constant surface concentration is extremely conservative and, as will become apparent in the following sections, masks many important aspects of the problem of pollutant migration through a finite layer.

If the clay liner (which may be natural or manmade) is underlain at some finite depth H by a far more permeable stratum with ground-water flow in the horizontal direction at an apparent (superficial, Darcy) velocity v_b , then solute will be transported away from the landfill at a rate dependent on the velocity, porosity and geometric dimensions of this layer. The concentration in the underlying more permeable stratum will only tend to zero when the base velocity is sufficiently large to remove the discharged leachate. However, in many cases the velocity will be relatively small and there will be a change in concentration with time in this stratum. To provide a means of estimating this concentration, it will be assumed that the concentration $c_b(t)$ in the underlying permeable stratum does not vary with vertical or horizontal position and that solute transport in this layer is only by advection (this is not strictly true and c_b really represents an average value beneath the landfill. This approximation is considered to be adequate for many practical situations; the more general two-dimensional case where the concentration varies with lateral position beneath the landfill will be considered in a subsequent article).

Consider a landfill with geometry as indicated in Fig. 1. The net change in the mass m of solute in the saturated permeable stratum beneath the landfill at any time t will be equal to the difference between the flux into the permeable stratum from the landfill and the flux from beneath the landfill due to flow in this stratum given by

$$m = \int_0^t WL f_b(c, \tau) d\tau - \int_0^t Wh v_b c d\tau \dots\dots\dots (6)$$

in which $f_b(c, \tau)$ = the flux into the permeable base stratum at $z = H$ and is given by Eq. 1; and W and L = the width and length of the landfill, respectively (the length is the average dimension of the landfill in the direction parallel to the velocity in the underlying permeable layer). It is implicitly assumed here that the permeable stratum is confined by a lower impermeable boundary and so the volume of water in this permeable stratum, beneath the landfill, depends on its thickness, h (thus, the assumption that c_b is independent of position is likely to be most appropriate when h is no more than a few meters). It follows from Eq. 6 that the corresponding base concentration c at time t is given by

$$c = \int_0^t \frac{f_b(c, \tau)}{n_b h} d\tau - \int_0^t \frac{v_b c}{n_b L} d\tau; \quad z = H \dots\dots\dots (7)$$

General Solution.—If we introduce a Laplace transform

$$\bar{c} = \int_0^\infty \exp(-st)c(t)dt \dots\dots\dots (8)$$

into Eqs. 4, 5 and 7, the governing Eq. 4 reduces to

$$s\bar{c} = -E \frac{\partial \bar{f}}{\partial z} \dots\dots\dots (9a)$$

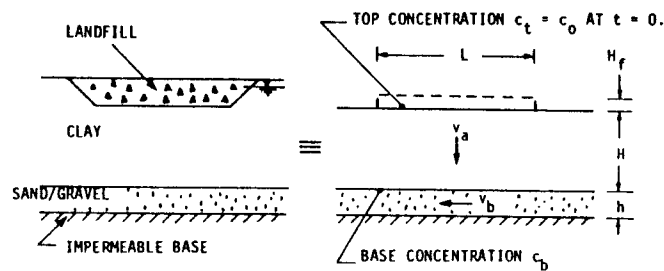


FIG. 1.—Problem Description

$$\text{in which } \bar{f} = n \left(\bar{c}v - D \frac{\partial \bar{c}}{\partial z} \right) \dots \dots \dots (9b)$$

$$\text{and } E = \frac{1}{n + \rho K} \dots \dots \dots (9c)$$

which must be solved subject to the transformed boundary conditions:

$$\bar{c} = \frac{c_0}{s} - \frac{\bar{f}}{sH_f} \text{ at } z = 0 \dots \dots \dots (10)$$

$$\bar{c} = \frac{\bar{f}}{sn_b h} - \frac{v_b \bar{c}}{sn_b L} \text{ at } z = H \dots \dots \dots (11)$$

Considering solutions of the form

$$\bar{c} = A \exp(kz) \dots \dots \dots (12a)$$

$$\bar{f} = B \exp(kz) \dots \dots \dots (12b)$$

then from Eq. 9,

$$\begin{bmatrix} s & kE \\ n(v - kD) & -1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0 \dots \dots \dots (13)$$

and since the determinant must be zero,

$$k^2 - \frac{v}{D}k - \frac{s}{nED} = 0 \dots \dots \dots (14)$$

This equation has roots α, β

$$\alpha = \frac{v}{2D} + \sqrt{\left(\frac{v^2}{4D^2} + \frac{s}{nED} \right)} \dots \dots \dots (15a)$$

$$\beta = \frac{v}{2D} - \sqrt{\left(\frac{v^2}{4D^2} + \frac{s}{nED} \right)} \dots \dots \dots (15b)$$

$$\text{Thus } \bar{c} = P \exp(\alpha z) + Q \exp(\beta z) \dots \dots \dots (16a)$$

$$\text{and } \bar{f} = nD\beta P \exp(\alpha z) + nD\alpha Q \exp(\beta z) \dots \dots \dots (16b)$$

Substituting Eq. 16 into Eqs. 10 and 11 and evaluating at $z = 0$ and $z = H$, respectively, gives

$$\begin{bmatrix} \left(1 + \frac{nD\beta}{sH_f}\right) & \left(1 + \frac{nD\alpha}{sH_f}\right) \\ \left(1 - \frac{nD\beta}{sn_b h} + \frac{v_b}{sn_b L}\right) \exp(\alpha H) & \left(1 - \frac{nD\alpha}{sn_b h} + \frac{v_b}{sn_b L}\right) \exp(\beta H) \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} \frac{c_o}{s} \\ 0 \end{bmatrix} \dots\dots\dots (17)$$

which can be solved for the constants P and Q .

Eqs. 15–17 define the solution at any depth z in transformed space. This solution can be numerically inverted for any time t using the technique proposed by Talbot (24).

These equations can be inverted analytically for a number of limiting cases as shown in Appendix I.

RESULTS

The parameters governing the migration of pollutants through clayey soils and clay liners vary significantly depending upon the nature of any particular site and species of pollutant. Although the theory is amenable to considering the entire spectrum of possible combinations of parameters, it is not possible to do so in this article. Thus, to illustrate the significant factors arising from the use of this formulation, consideration will be given to the hypothetical case of a 200 m long (see Fig. 1) landfill resting on a homogeneous clay stratum with porosity $n = 0.4$ underlain by a relatively permeable, 1 m thick, sand stratum ($h = 1$ m) with porosity $n_b = 0.3$. The dispersion coefficient will be taken to be $D = 0.01$ m²/y. Unless otherwise noted, the clay layer is assumed to be 2 m thick ($H = 2$ m), the apparent advection velocity, v_a , is zero and there is no geochemical reaction ($pK = 0$).

Effect of Leachate Height and Base Velocity.—The total mass of any particular species of pollutant available for migration from the landfill is directly related to the concentration of that species within the leachate and to the total volume of leachate. The concentration can be measured. It is more difficult to determine the volume of contaminant in the landfill. However, it is possible to make estimates of likely upper and lower bounds for the volume of leachate by considering the porosity of the landfill material and the height of the watertable within the landfill. The height of leachate H_f represents the volume of leachate divided by the area of the landfill. An examination of a limited number of landfills would suggest that H_f is likely to range from 0.5–10 m, with values of 1–5 m being most probable (based on 8, 15, and other unpublished reports). H_f is related to the water content of the landfill material and, with the exception of lagoons, will be less than the total height of fill material.

Recognizing that contaminant will eventually diffuse through clay liners into any underlying ground water (even if there is no advection), the design and certification of landfills should involve consideration of

the maximum expected concentration of contaminant within the ground water. Often, the initial concentration of pollutant within the leachate will be up to several orders of magnitude higher than recommended levels for potable waters and the designer will be concerned with maintaining quite low concentrations at the base of the liner for a considerable time after emplacement, e.g., the leachate concentrations of chloride and phenol in a municipal landfill at Sarnia, Ontario (Crooks and Quigley, personal communication) are 3,000 mg/L and 0.150 mg/L, respectively whereas recommended limits for these in drinking water are 250 mg/L and 0.001 mg/L, respectively (25).

The effect of the height of leachate H_l upon the surface concentrations for the limiting cases where the base concentration c_b is zero (i.e., the base velocity approaches infinity) and the base velocity is zero are shown in Figs. 2(a and b), respectively. With $c_b = 0$ (infinite base velocity), the surface concentration decreases fastest for low values of H_l but in all cases tends to zero in the steady state. In the second limiting case, with zero base velocity, the surface concentration decreases until it reaches a steady-state value (which depends upon H_l), and the base concentration increases monotonically with time until it attains a steady-state value equal to the surface concentration.

The situation where there is zero base velocity provides the worst-case condition for a given value of H_l . In most practical situations where a landfill is underlain by a permeable stratum, there will be some average, nonzero base velocity v_b . From the results given in Fig. 2, it may be anticipated that for this case the base concentration will increase with time reaching a maximum value at some time t_{max} and then will decrease until the steady state of zero concentration is reached. This behavior is shown in Fig. 3 for $H_l = 1$ m and a number of apparent base velocities v_b .

Similar trends are observed for all values of H_l except for the situation where H_l tends to infinity. This limiting case corresponds to a constant surface concentration and is fundamentally different from that for any finite value of H_l . Here, the base concentration increases with time and becomes asymptotic to a steady-state value which depends on the base velocity. This case represents the upper limit in terms of both the maximum base concentration and the time required to reach the maximum value. This situation is unlikely to occur in practice.

Figs. 4(a-b) summarize the effect of base velocity upon the maximum base concentration and the time required to achieve this maximum for a range of values of H_l . As might be expected, increasing the base velocity results in greater dilution of contaminant; thus, the peak concentration is lowered and is reached at an earlier time. For moderate to large velocities (i.e., $v_b > 10$ m/y) the maximum base concentration varies inversely with the base velocity whereas the time required to reach this maximum is relatively independent of velocity. This observation allows convenient extrapolation for velocities greater than those shown on Fig. 4.

In summary, it is noted that for a finite mass of contaminant and nonzero base velocity, the concentration of pollutant in the ground water beneath the liner will reach a maximum value c_b at some definite time t_{max} and will decrease for greater times. This maximum value can be used in design to ensure that contamination of the ground water never exceeds a specified level. In this context, the design of the liner can be

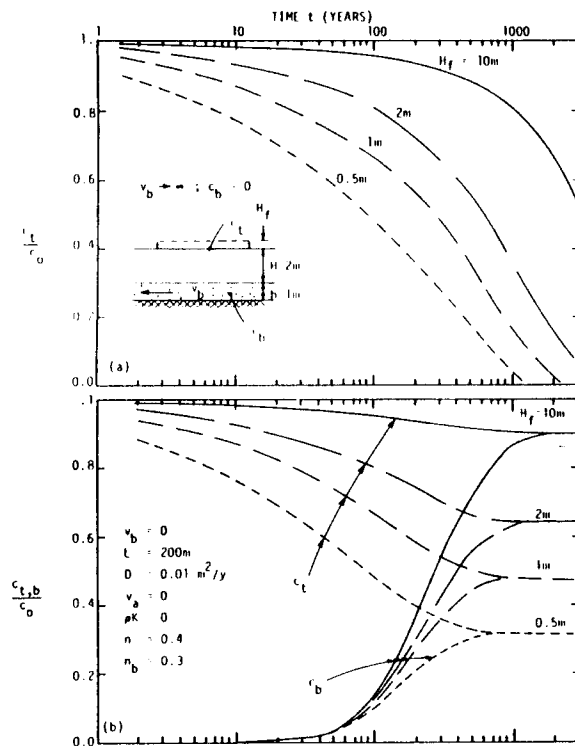


FIG. 2.—Variation in Concentration With Time for Different Heights of Leachate H_f : (a) Base Concentration Zero ($v_b \rightarrow \infty$); (b) Base Velocity Zero ($v_b = 0$)

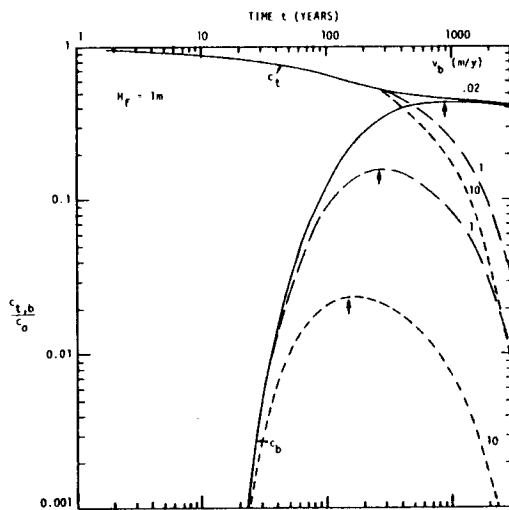


FIG. 3.—Variation in Top and Base Concentrations With Time for Different Base Velocities v_b ; $H_f = 1m$

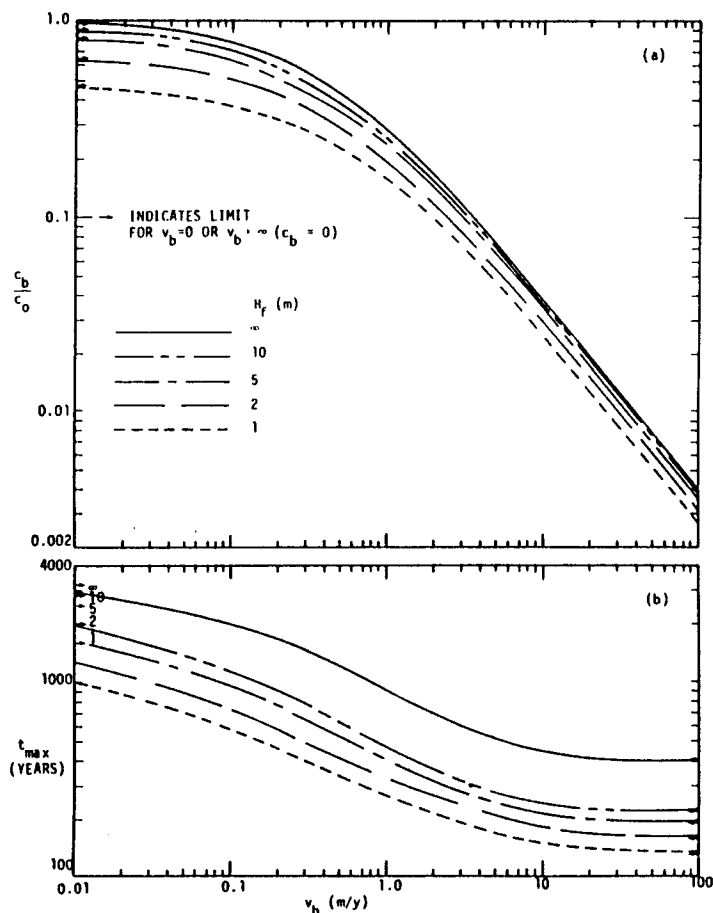


FIG. 4.—Effect of Base Velocity v_b Upon: (a) Maximum Base Concentration c_b ; (b) Time Required to Reach Maximum Concentration, t_{max}

optimized by considering a realistic value of leachate height and base velocity, rather than highly conservative limiting values.

Effect of Geochemical Reaction.—The absorption process is most easily represented in terms of the dimensionless quantity ρK . The value of ρK will depend upon the soil properties, chemical reactions and their rates, and the range of concentration. Typical values lie between 0 and 100 although much higher values have been reported (e.g., see 6, 7, 10–12, 14, 22).

If, as is usually assumed, the surface concentration is constant ($H_f = \infty$) then the effect of the geochemical reaction is simply to slow down the dispersion-advection process. Thus, the maximum base concentration is independent of the geochemical reaction although the time t_{max} required to reach this maximum does increase significantly. This is not true when H_f is finite.

In the previous section, it was shown that the effect of the height of

leachate being finite was to reduce the maximum base concentration. This effect is greatly increased by any geochemical reaction. Fig. 5 shows the variation in both the top and bottom concentration with time for the case where there is no sorption ($\rho K = 0$) and where there is moderate sorption ($\rho K = 10$). In both cases, the base concentration increases with time until it reaches a maximum value and then decreases. As might be expected, the geochemical reaction increases the time required to reach the peak concentration by approximately one order of magnitude for $\rho K = 10$. Of greater interest, in comparison with the situation for $H_f = \infty$, is the fact that the maximum base concentration is also reduced by approximately one order of magnitude.

At small to moderate times, the sorption process results in lower surface concentrations for $\rho K = 10$ than for $\rho K = 0$. At times greater than that required to reach the maximum concentration for $\rho K = 10$, the surface concentration for $\rho K = 10$ exceeds that for lower values of ρK . The reason for this is that the clay acts as a buffer. At small to moderate times, the clay absorbs pollutant from the pore fluid thereby reducing both the concentration and the concentration gradient. Each position in the clay will reach a maximum concentration at some point in time. Once the time has been reached, the clay will cease sorbing pollutant. Depending on the details of the particular sorption process involved, the contaminant sorbed onto the clay will either remain fixed to the clay or in the worst case, as assumed here, will desorb back into the pore fluid. It is this release which is responsible for the higher values of concentration throughout the deposit for $\rho K = 10$ than for $\rho K = 0$ at large times. This buffering role of the clay only becomes apparent if the finite mass

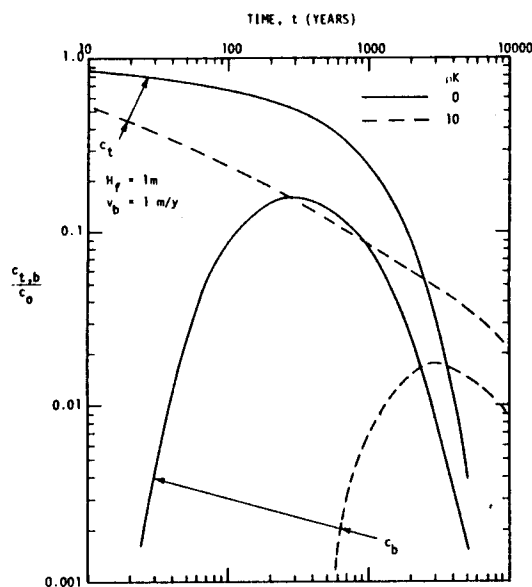


FIG. 5.—Effect of Geochemical Reaction Upon Variation of Top and Base Concentration With Time: $H_f = 1$ m

of pollutant in the landfill is considered.

Figs. 6 and 7 show the effect of geochemical reaction upon the maximum base concentration and the time required to reach this maximum for $v_b = 1$ m/y. For $\rho K < 0.1$, the geochemical reaction has negligible

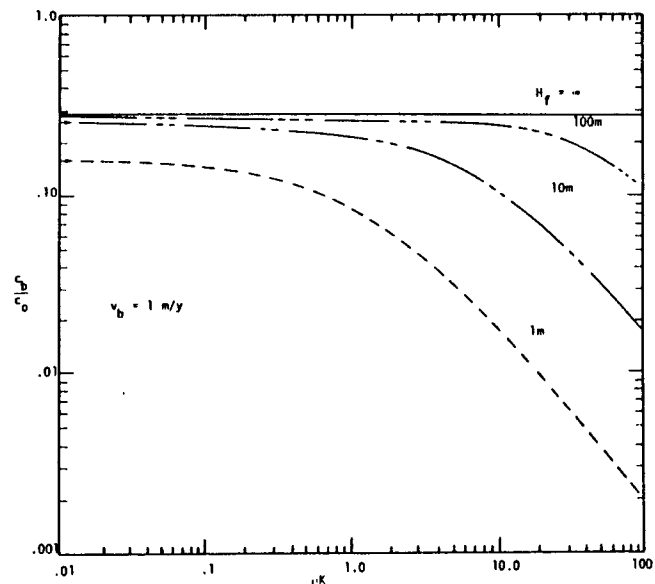


FIG. 6.—Effect of Sorption Upon Maximum Base Concentration: $v_b = 1$ m/y

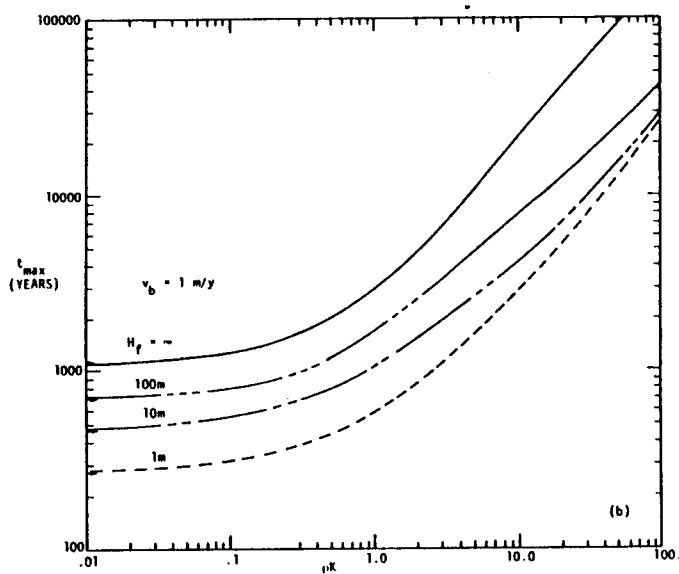


FIG. 7.—Effect of Sorption Upon Time to Reach Maximum Concentration: $v_b = 1$ m/y

effect upon the base concentration. If $\rho K > 1.0$, then the geochemical reaction significantly decreases the base concentration for most practical cases (i.e., $H_f \leq 10$ m) and should be considered in design. The effect of the geochemical reaction is most pronounced for small heights of leachate (H_f) but may still warrant consideration for quite large heights ($H_f < 100$ m) if the geochemical reaction is strong (i.e., $\rho K > 10$). For high values of ρK , the base concentration varies inversely with ρK .

The time required to reach the maximum base concentration (see Fig. 7) increases for $\rho K > 1.0$ and approaches a linear relationship for high values of ρK . Also of interest is the fact that the relationship between

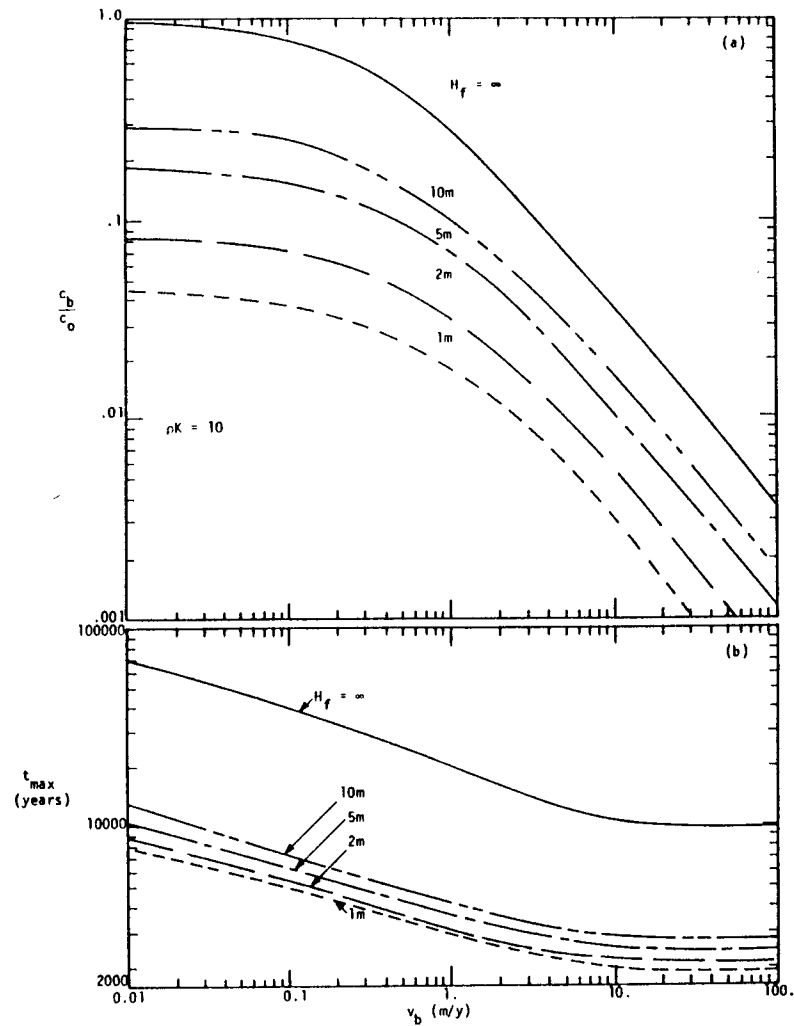


FIG. 8.—Effect of Base Velocity v_b Upon: (a) Maximum Base Concentration c_b ; (b) Time Required to Reach Maximum Concentration t_{max} ; $\rho K = 10$

$t_{\max}$ and ρK tends to a single curve for $H_f < \infty$ at very large values of ρK .

The effect of base velocity v_b is shown in Fig. 8 for $\rho K = 10$ and may be compared with the results given in Fig. 4 for $\rho K = 0$. The same general trends are evident in both figures. The effect of base velocity upon base concentration can generally be neglected (to an accuracy of better than 20%) for $v_b < 0.1$ m/y although it should be noted even these small base velocities may reduce the value of $t_{\max}$ by up to a factor of 2. For $v_b > 10$ m/y, the relationship between base concentration and base velocity tends to the inverse relation whereas time $t_{\max}$ is relatively independent of v_b . Of particular importance is the fact that the geochemical reaction greatly enhances the effect of H_f . For $\rho K = 0$, the value of c_b varies by up to a factor of 2.2 for H_f in the range $1 \leq H_f \leq \infty$. For $\rho K = 10$, the corresponding variation in c_b is up to a factor of 22.

Effect of Layer Thickness.—In design, the thickness and known attenuation potential (ρK) of the clay layer isolating the landfill from the underlying ground-water system may be used to control the maximum base concentration (e.g., Ref. 2). Figs. 9–12 show the variation in c_b and $t_{\max}$ with clay liner thickness H for $\rho K = 0$ and $\rho K = 10$. Increasing the layer thickness substantially reduces the maximum concentration ever reached at the base and quite obviously increases the time required to reach this maximum. For example, with $\rho K = 0$, increasing the clay liner thickness from 0.5 m to 4 m decreases the $c_{b(\max)}$ by up to an order of magnitude (see Fig. 13) and increases the time $t_{\max}$ by up to an order of magnitude.

The effect of layer thickness is greatly increased by consideration of the height of leachate and the geochemical reaction. Thus, the thickness of liner required to ensure that a specified maximum base concentration is never exceeded may be significantly reduced by considering the interaction of all of these factors.

Effect of Advection Velocity.—The advection velocity v_a within the clay deposit will depend on the hydraulic conductivity of the clay and the difference between the head in the landfill and the head in the underlying aquifer. For the 1-D conditions assumed in this paper, v_a can be readily calculated from a knowledge of these quantities. If, as is often the case, the head in the landfill is greater than in the aquifer, the velocity v_a is positive and the theory used in this paper can be used directly. If the head in the landfill is less than in the aquifer, there will be upward flow into the landfill. With minor modifications to the boundary conditions, this case can also be examined. It should be noted that even with upward flow, diffusion through the clay liners may still occur although clearly, the rate of mass transfer will be much smaller than when there is downward flow.

All the previous results were obtained for the limiting case of zero advection. Whenever possible, a disposal site should be located such that there is very little seepage through the waste into the surrounding soil and local ground-water system. If the disposal site must be located in a region where there is likely to be significant seepage, then the seepage through the waste may be minimized by containing the waste within a clay liner which is itself fully surrounded by a far more permeable graded filter which effectively creates an equipotential surface around the liner (e.g., see Ref. 18).

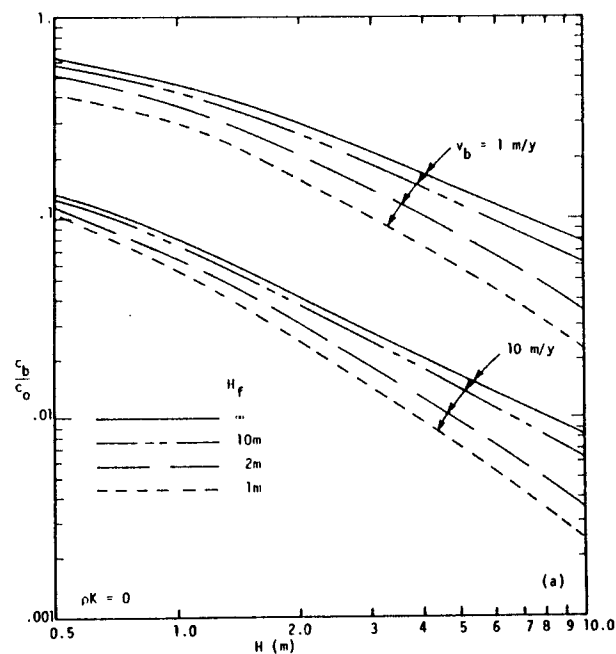


FIG. 9.—Effect of Layer Thickness Upon Maximum Base Concentration: $\rho K = 0$

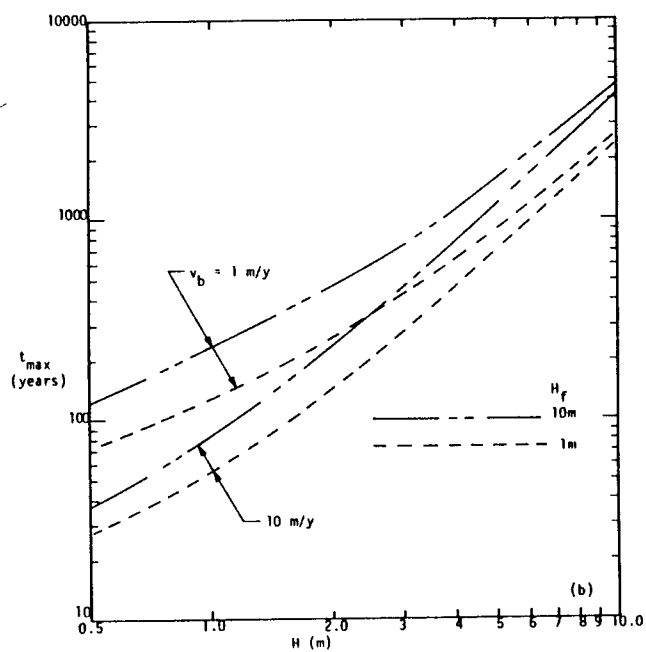


FIG. 10.—Effect of Layer Thickness Upon Time to Reach Maximum Concentration: $\rho K = 0$

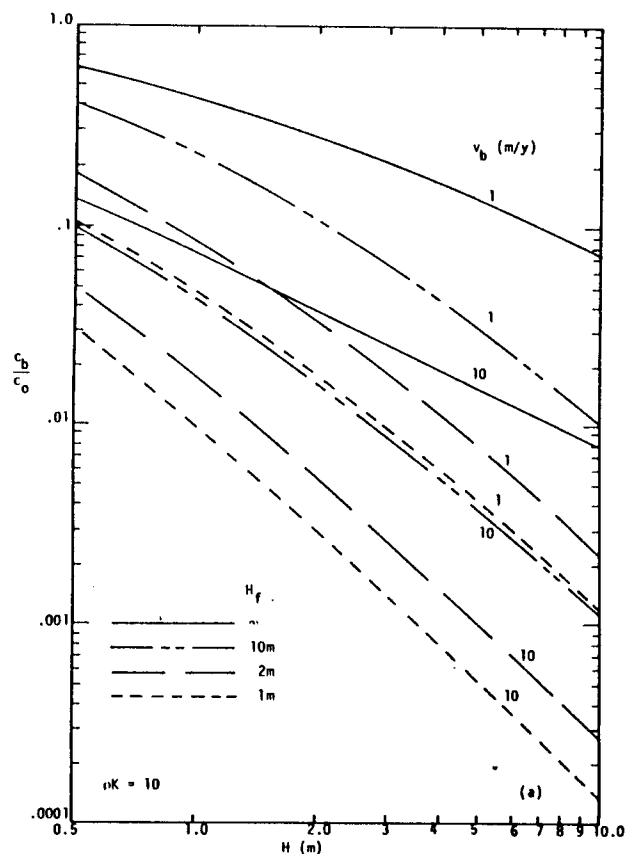


FIG. 11.—Effect of Layer Thickness Upon Maximum Base Concentration, $\rho K = 10$

Although seepage may be minimized, it will never be precisely zero and it is of interest to examine the effect of the apparent (superficial) advective velocity through the liner upon the maximum base concentration. Analyses were performed for a range of advection velocities. The results exhibited the same trends with regard to the effect of base velocity, layer thickness, leachate height, and geochemical reaction as reported in the previous section. However, the advection velocity did increase the maximum base concentration and decrease the time required to reach this concentration as shown for a 2 m thick liner with $\rho K = 0$ in Fig. 13 (the results for other values of ρK are very similar except that the values of c_b and $t_{\max}$ differ in approximately the same ratio as they do for $v_a = 0$ in Figs. 6–8).

From Fig. 13, it can be seen that the advective velocity could be neglected for $v_a < 0.0004$ m/y. For v_a up to 0.001 m/y, advection increases base concentration by up to 33% although $t_{\max}$ is almost constant. Advective velocity greater than 0.001 m/y may significantly affect the base concentration, with the values of $v_a = 0.01$ m/y being up to 5 times greater than those for $v_a = 0$. The time $t_{\max}$ is somewhat less affected by

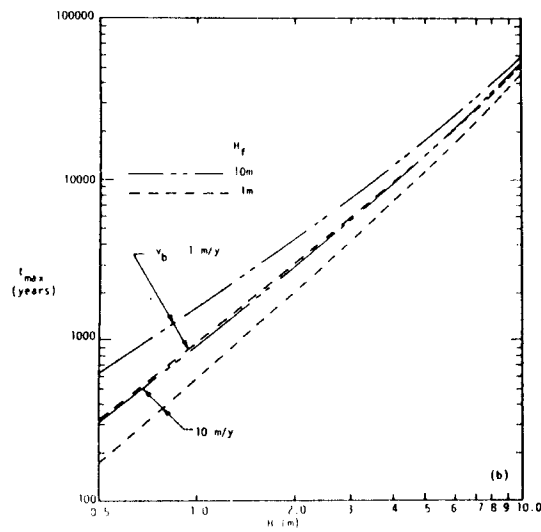


FIG. 12.—Effect of Layer Thickness Upon Time to Reach Maximum Concentration: $\rho K = 10$

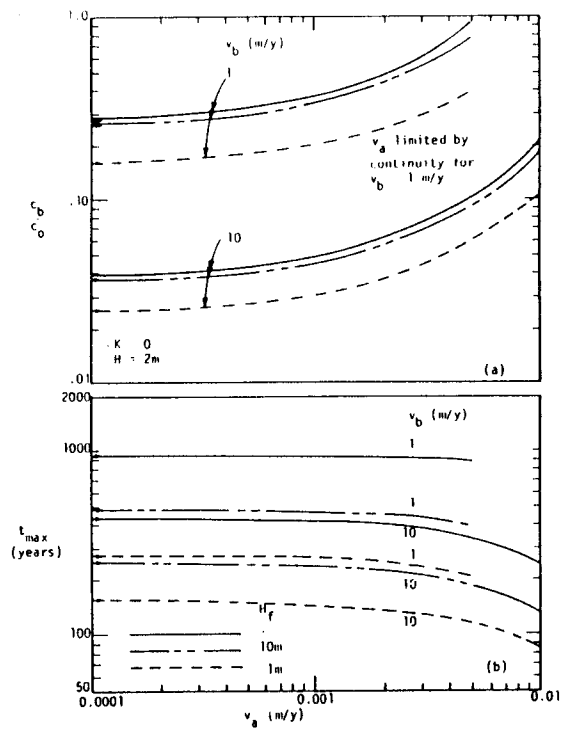


FIG. 13.—Effect of Advection Velocity v_a Upon: (a) Maximum Base Concentration c_b ; (b) Time Required to Reach Maximum Concentration t_{max}

v_b , varying by less than a factor of 2 for the same range in advective velocity. This demonstrates the importance of minimizing the advective velocity in the siting and design of the waste disposal site.

APPLICATION

The techniques developed in the theory section can be readily programmed and have been implemented in the program POLLUTE (20). This program involves minimal data preparation and computational effort.

The analysis is suitable for use in situations where the "liner" inhibiting contamination migration is relatively homogeneous and the thickness of the liner is small compared to the dimensions of the landfill (i.e., where H/L is less than 0.05). The analysis can be readily extended to multilayered 2-D and 3-D situations involving single or multiple interacting species and this will be described in subsequent articles. However, it is considered that the simple 1-D case examined here will be adequate for a large range of practical situations. The technique is currently being used in conjunction with a detailed field study of a landfill site at Sarnia, Ontario.

CONCLUSIONS

A technique for the analysis of 1-D pollutant migration through a soil layer of finite depth has been presented. This formulation permits consideration of the depletion of contaminant in the landfill with time as well as the effect of a flushing velocity in a permeable stratum beneath the clay layer. The technique requires minimal data preparation, is accurate, and is computationally very efficient compared with alternative techniques such as finite element and finite difference methods (e.g., Ref. 23 and many others).

A limited parametric study was performed to demonstrate some of the important features which arise from considering the finite mass of contaminant and a flushing velocity at the base of the clay liner. For a finite mass of contaminant and the range of parameters considered, the following may be concluded:

1. If the base velocity is greater than zero, then the concentration of pollutant in the ground water beneath the liner will reach a maximum value, c_b , at some time t_{max} , and will decrease for greater times. This maximum value can be used in design to ensure that the contamination of the ground water never exceeds a specified level (if the base velocity is zero then the base concentration reaches a maximum and remains at that value for all subsequent time).
2. The magnitude of the maximum base concentration decreases: (a) with decreasing mass of contaminant in the landfill; and (b) with increasing base velocity.
3. Geomechanical reactions (sorption) can greatly affect the magnitude of the maximum base concentration as well as the time required to reach this maximum. The sorption potential will depend on the species of contaminant being considered and the chemical properties of the clay.

For reactive species, the clay adsorbs pollutant when the concentration is increasing. Once the concentration at a point reaches a maximum value, the sorbed contaminant may remain fixed to the clay or, in the worst case, may be released back into the pore fluid as the concentration drops. This beneficial buffering role for some species of pollutant is only apparent if the finite mass of pollutant in the landfill is considered.

4. The effect of sorption is greatest for low to moderate volumes of leachate (i.e., leachate height less than 10 m). Sorption may be particularly important in designing for hazardous substances such as NH_4 and heavy metals although careful consideration of the chemical properties of the clay is necessary.

5. The maximum base concentration can be decreased by increasing clay liner thickness. However, the design of the liner may be optimized by also considering the effects of leachate height, base velocity and any geochemical reaction.

6. The consideration of advection velocity does not alter the trends described above, but it does increase the maximum base concentration and decrease the time required to reach this concentration.

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APPENDIX I.—SOLUTION OF GOVERNING EQUATIONS FOR LIMITING CASES

As the volume of the landfill tends to infinity, the surface concentration will tend to a constant value c_0 . Also taking the limit as the depth H tends to infinity, Eq. 17 yields

$$P = 0, \quad Q = \frac{c_0}{s}$$

$$\text{thus } \bar{c} = \frac{c_0}{s} \exp(\beta z) \dots\dots\dots (18a)$$

$$\text{in which } \beta z = \frac{vz}{2D} - a\sqrt{s + b^2} \dots\dots\dots (18b)$$

$$\text{and } a = \frac{z}{\sqrt{nED}} \dots\dots\dots (19a)$$

$$b^2 = \frac{nEv^2}{4D} \dots\dots\dots (19b)$$

Eqs. 18 can be inverted analytically yielding $c = J(z, t)$ in which

$$J(z, t) = \frac{c_0}{2} \exp\left(\frac{vz}{2D}\right) \left\{ \exp(-ab) \operatorname{Erfc}\left(\frac{a}{2\sqrt{t}} - b\sqrt{t}\right) + \exp(ab) \operatorname{Erfc}\left(\frac{a}{2\sqrt{t}} + b\sqrt{t}\right) \right\} \dots\dots\dots (20)$$

which corresponds to the classic solution (e.g., Ref. 19).

Considering now the limiting case as the surface concentration tends to a constant c_0 but the layer depth H is finite, being underlain by a permeable stratum which is fully flushed such as $c = 0$ at $z = H$. Then

$$P + Q = \frac{c_0}{s} \dots\dots\dots (21a)$$

$$P \exp(\alpha H) + Q \exp(\beta H) = 0 \dots\dots\dots (21b)$$

Solving for P and Q and substituting into Eq. 16a yields

$$\begin{aligned} \bar{c} &= \frac{c_0}{s} \left[\frac{\exp(\alpha z + \beta H) - \exp(\beta z + \alpha H)}{\exp(\beta H) - \exp(\alpha H)} \right] \\ &= -\frac{c_0}{s} \left[\frac{\exp(\alpha z + \beta H) - \exp(\beta z + \alpha H)}{\exp(\alpha H)} \right] \sum_{p=0}^{\infty} \exp[(\beta - \alpha)pH] \dots\dots\dots (22) \end{aligned}$$

Therefore

$$\begin{aligned} \bar{c} &= -\frac{c_0}{s} \left\{ \sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}[(p+1)H - z]\right) \exp[\beta(2(p+1)H - z)] \right. \\ &\quad \left. - \sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}pH\right) \exp(2pH + z)\beta \right\} \dots\dots\dots (23) \end{aligned}$$

Now, as indicated earlier, an equation of the form $\bar{c} = c_0/s \exp(\beta z)$ has the solution $c = J(z, t)$ in which $J(z, t)$ is defined by Eq. 19 for any z, t and thus, Eq. 20 may be inverted to give

$$\begin{aligned} c &= -\left\{ \sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}[(p+1)H - z]\right) J[2(p+1)H - z, t] \right. \\ &\quad \left. - \sum_{p=0}^{\infty} \exp\left(-\frac{v}{D}pH\right) J(2pH + z, t) \right\} \dots\dots\dots (24) \end{aligned}$$

APPENDIX II.—REFERENCES

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